JEE Advanced 2026

Sample Paper - 5 (Paper-2)

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and three sub-sections as below.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

MRQ

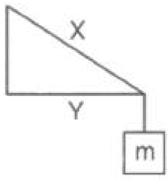
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

Physics

1. Two rods X and Y are attached to a weight of mass M as shown in figure, then: **[3]**



a) both X and Y experience compression

b) Y experiences extension and X compression

c) X experiences extension and Y compression

d) both X and Y experience extension

2. Two coils of self-inductance L_1 and L_2 are placed so close to each other that the effective flux in one coil is completely linked with the other; then the mutual inductance M between them is given by: **[3]**

a) $M = \frac{L_1}{L_2}$

b) $M = L_1 + L_2$

c) $M = L_1 - L_2$

d) $M = \sqrt{L_1 L_2}$

3. When you make ice cubes, the entropy of water: **[3]**

a) increases

b) does not change

c) may either increase or decrease depending on the process used

d) decreases

4. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d (d << l) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result, the charges approach each other with a velocity v. Then, as a function of distance x between them, is: **[3]**

a) $v \propto x$

b) $v \propto x^{-1/2}$

c) $v \propto x^{-1}$

d) $v \propto x^{1/2}$

5. Holes are charge carriers in **[4]**

a) p-type semiconductors

b) metals

c) ionic solids

d) intrinsic semiconductors

6. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true? **[4]**

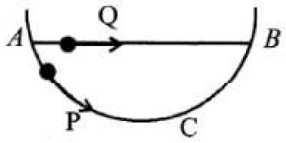
a) The filament consumes less electrical power towards the end of the life of the bulb

b) The temperature distribution over the filament is uniform

c) The filament emits more light at higher band of frequencies before it breaks up

d) The resistance over small sections of the filament decreases with time

7. A particle P is sliding down a frictionless hemispherical bowl. It passes the points at t = 0. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at t = 0 along the horizontal string AB, with the speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B. [4]



a) $t_P > t_Q$

b) $t_P < t_Q$

c) $t_P = t_Q$

d) $\frac{t_P}{t_Q} = \frac{\text{length of arc } ACB}{\text{length of arc } AB}$

8. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at x = 0 and positive plate is at x = 3d. The slab is equidistant from the plates. The capacitor is given some charge. As one goes from 0 to 3d, [4]

a) the electric potential increases at first, then decreases and again increases.

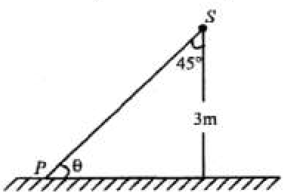
b) the direction of the electric field remains the same.

c) the magnitude of the electric field remains the same.

d) the electric potential increases continuously

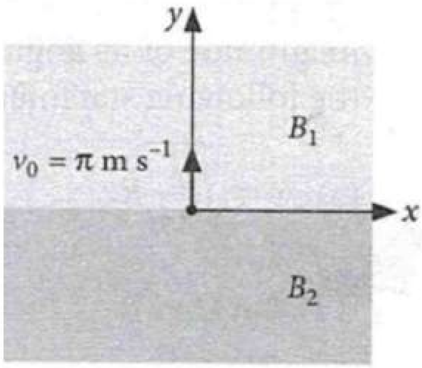
9. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways-engineer uses dimensions and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is [4]

10. Spotlight S rotates in a horizontal plane with constant angular velocity of 0.1 radian/second. The spot of light P moves along the wall at a distance of 3 m. The velocity of the spot P when $\theta = 45^o$ (see fig.) is ________ m/s [4]



11. In the xy-plane, the region y > 0 has a uniform magnetic field $B_1\hat{k}$ and the region y < 0 has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi ms^{-1}$ at t = 0, [4]

as shown in the figure. Neglect gravity in this problem. Let t = T be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$ the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is ________.



12. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad/s $^{-1}$ about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is **[4]**

13. A projectile is thrown from a point O on the ground at an angle 45^O from the vertical and with a speed 5 m/s. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity g = 10 m/s^2. The value of x is ________. **[4]**

14. Two inductors L_1 (inductance 1 mH, internal resistance 3Ω) and L_2 (inductance 2 mH, internal resistance 4Ω), and a resistor R (resistance 12Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is **[4]**

15. A ball is thrown from the location $(x_0, y_0) = (0, 0)$ of a horizontal playground with an initial speed v_0 at an angle θ_0 from the $+x$-direction. The ball is to be hit by a stone, which is thrown at the same time from the location $(x_1, y_1) = (L, 0)$. The stone is thrown at an angle $(180 - \theta_1)$ from the $+x$-direction with a suitable initial speed. For a fixed v_0, when $(\theta_0, \theta_1) = (45^\circ, 45^\circ)$, the stone hits the ball after time T_1, and when $(\theta_0, \theta_1) = (60^\circ, 30^\circ)$, it hits the ball after time T_2. In such a case, $(T_1/T_2)^2$ is ________. **[4]**

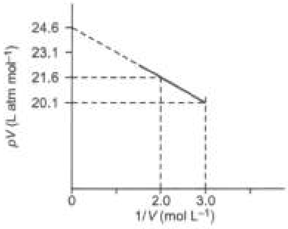
16. A drop of liquid of radius R = 10^{-2} m having surface tension $S = \frac{0.1}{4\pi} Nm^{-1}$ divides itself into K identical drops in this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$, then the value of α is : **[4]**

Chemistry

17. The gas X at 1 atm is bubbled through a solution containing a mixture of 1 M Y^- and 1 M Z^- at 25°C. If the order of reduction potential is Z > Y > X , then **[3]**

a) Y will oxidise Z and not X
b) Y will oxidise X and not Z
c) Y will reduce both X and Z
d) Y will oxidise both X and Z

18. For one mole of a van der Waals' gas when b = 0 and T = 300 K, the pV vs 1/V plot is shown below. The value of the van der Waals' constant a (atm L^2 mol^{-2}) is **[3]**



a) 1.5
b) 1.0
c) 4.5
d) 3.0

19. For which change $\Delta H \neq \Delta E$: **[3]**

a) $HCl + NaOH \rightarrow NaCl$
b) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$
c) $C(s) + O_2(g) \rightarrow CO_2(g)$
d) $H_2(g) + I_2(g) \rightarrow 2HI(g)$

20. In the Cannizzaro reaction given below, **[3]**

$2PhCHO \xrightarrow{OH^-} PhCH_2OH + PhCO_2^-$, the slowest step is

a) the transfer of hydride to the carbonyl group
b) the deprotonation of $PhCH_2OH$
c) the abstraction of proton from the carboxylic acid
d) the attack of -OH at the carbonyl group

21. The pair(s) of coordination complexes/ions exhibiting the same kind of isomerism is(are) **[4]**

a) $[Co(NH_3)_4Cl_2]^+$ and $[Pt(NH_3)_2(H_2O)Cl]^+$

b) $[Cr(NH_3)_5Cl]Cl_2$ and $[Cr(NH_3)_4Cl_2]Cl$

c) $[Pt(NH_3)_3(NO_3)]Cl$ and $[Pt(NH_3)_3Cl]Br$

d) $[CoBr_2Cl_2]^{2-}$ and $[PtBr_2Cl_2]^{2-}$

22. Benzene and naphthalene form an ideal solution at room temperature. For this process, the true statement(s) is(are) **[4]**

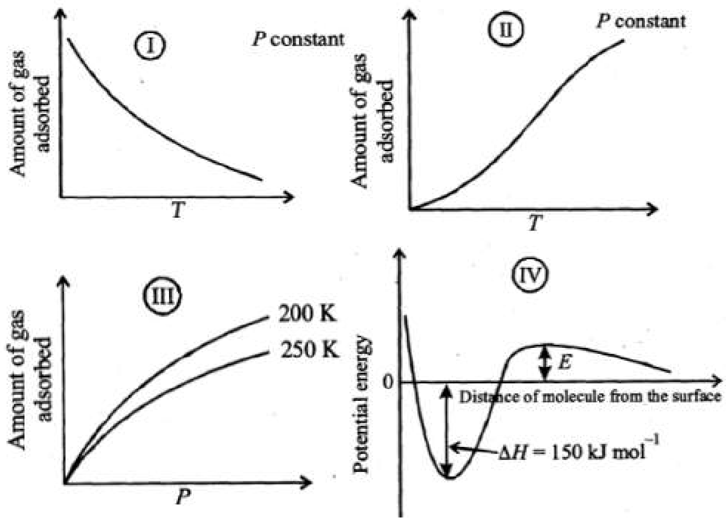
a) $\Delta H = 0$

b) $\Delta S_{\text{surroundings}} = 0$

c) ΔS_{system} is positive

d) ΔG is positive

23. The given graphs/data I, II, III and IV represent general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice(s) about I, II, III and IV is (are) correct? **[4]**



a) I is physisorption and II is chemisorption

b) I is physisorption and III is chemisorption

c) IV is chemisorption and II is chemisorption

d) IV is chemisorption and IE is chemisorption

24. For diatomic molecules, the correct statement(s) about the molecular orbitals formed by the overlap of two $2p_z$ orbitals is (are) **[4]**

a) σ^* orbital has one node in the xz-plane containing the molecular axis.

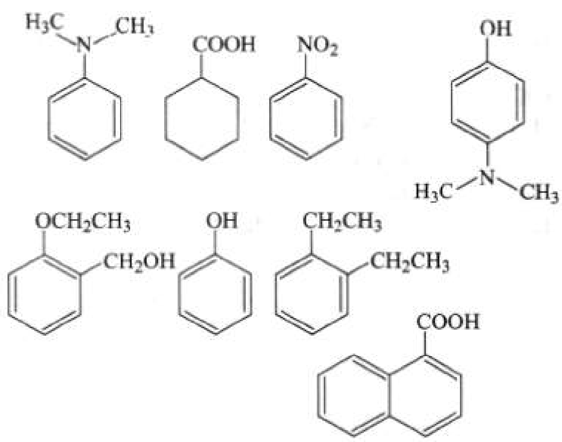
b) σ orbital has a total of two nodal planes.

c) π orbital has one node in the plane which is perpendicular to the molecular axis and goes through the center of the molecule.

d) π^* orbital has one node in the xy-plane containing the molecular axis.

25. The composition of a sample of Wurtzite is $Fe_{0.93}O_{1.00}$. What percentage of the iron is present in the form of Fe (III)? **[4]**

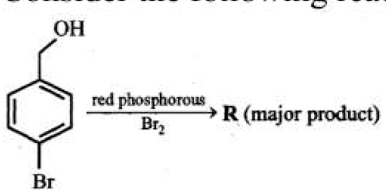
26. Amongst the following, the total number of compounds soluble in aqueous NaOH is **[4]**



27. Concentration of H_2SO_4 and Na_2SO_4 in a solution is 1 M and 1.8×10^{-2} M, respectively. Molar solubility of $PbSO_4$ in the same solution is $X \times 10^{-Y}$ M (expressed in scientific notation). The value of Y is ________. **[4]**
[Given: Solubility product of $PbSO_4(K_{sp}) = 1.6 \times 10^{-8}$. For H_2SO_4, K_{a1} is very large and $K_{a2} = 1.2 \times 10^{-2}$]

28. A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is ________ min. (Round off to the nearest integer). **[4]**
[Use :ln 2 = 0.69; In 10 = 2.3]

29. Consider the following reaction. **[4]**



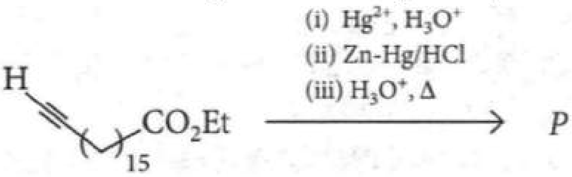
On estimation of bromine in 1.00 g of R using Carius method, the amount of AgBr formed (in g) is________. [Given: Atomic mass of H = 1, C = 12,0 = 16, P = 31, Br = 80, Ag = 108]

30. For He^+, a transition takes place from the orbit of radius 105.8 pm to the orbit of radius 26.45 pm. The wavelength (in nm) of the emitted photon during the transition is ________. [4]

[Use: Bohr radius, a = 52.9pm; Rydberg constant, $R_H = 2.2 \times 10^{-18}$ J; Planck's constant, $h = 6.6 \times 10^{-34}$ J s; Speed of light, $c = 3 \times 10^8$ ms^{-1}]

31. Calculate the molality of 1 litre solution of 93% H_2SO_4 (weight/volume). The density of the solution is 1.84 g/mL. [4]

32. In the following reaction sequence, the major product P is formed. [4]



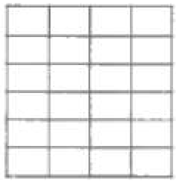
Glycerol reacts completely with excess P in the presence of an acid catalyst to form Q.

Reaction of Q with excess NaOH followed by the treatment with $CaCl_2$ yields Ca-soap R, quantitatively. Starting with one mole of Q, the amount of R produced in gram is ________.

[Given, atomic weight: H = 1, C = 12, N = 14, O = 16, Na = 23, Cl = 35, Ca = 40]

Mathematics

33. A rectangle with sides of length (2m - 1) and (2n - 1) units is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is [3]



a) $m(m+1)n(n+1)$ b) $(m+n-1)^2$

c) 4^{m+n-1} d) m^2n^2

34. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then [3]

a) $t_3 > t_1 > t_2 > t_4$ b) $t_1 > t_2 > t_3 > t_4$

c) $t_2 > t_3 > t_1 > t_4$ d) $t_4 > t_3 > t_1 > t_2$

35. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in: **[3]**

a) H.P
b) H.M.
c) G.P
d) A.P

36. The equation $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1, |r| < 1$ represents **[3]**

a) a circle
b) A parabola
c) a hyperbola
d) an ellipse

37. Let $f(x) = \frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$. Then **[4]**

a) $\lim_{x\to1^+} f(x)$ does not exist
b) $\lim_{x\to1^+} f(x) = 0$
c) $\lim_{x\to1^-} f(x)$ does not exist
d) $\lim_{x\to1^-} f(x) = 0$

38. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then **[4]**

a) there is 3 × 3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix
b) determinant of $(M^2 + MN^2)$ is 0
c) for a 3 × 3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix
d) determinant of $(M^2 + MN^2) \geq 1$

39. Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a n × n matrix with $P_{ij} = \omega^{i+j}$. Then $p^2 \neq 0$, when n = **[4]**

a) 58
b) 55
c) 56
d) 57

40. Let $\triangle PQR$ be a triangle. Let $\vec{a} = \overrightarrow{QR}, \vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12, |\vec{b}| = 4\sqrt{3}, \vec{b}.\vec{c} = 24$, then which of the following is (are) true? **[4]**

a) $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$
b) $\vec{a}.\vec{b} = -72$

c) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$ d) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$

41. Consider the lines L_1 and L_2 defined by **[4]**
$L_1 : x\sqrt{2} + y - 1 = 0$ and $L_2 : x\sqrt{2} - y + 1 = 0$
For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ_2. The line y = 2x + 1 meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.
Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the **square** of the distance between R' and S'. The value of D is ________.

42. The value of $\frac{48S_2}{\pi^2}$ is ________. **[4]**

43. For $x \in \mathbb{R}$, then number of real roots of the equation $3x^2 - 4\left|x^2 - 1\right| + x - 1 = 0$ is ________. **[4]**

44. Let S_k, where k = 1, 2, ..., 100, denotes the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then, the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} I\left(k^2 - 3k + 1\right) S_k$ is **[4]**

45. Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1. Let a be the area of the triangle ABC. Then the value of $(64a)^2$ is **[4]**

46. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is **[4]**

47. Three lines are given by $\vec{r} = \lambda\hat{i}, \lambda \in R$; $\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in$ R and $\vec{r} = v(\hat{i} + \hat{j} + \hat{k}), v \in R.$ **[4]**
Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals ________.

48. In a study about a pandemic, data of 900 persons was collected. It was found that **[4]**
190 persons had symptom of fever,
220 persons had symptom of cough,
220 persons had symptom of breathing problem,
330 persons had symptom of fever or cough or both,
350 persons had symptom of cough or breathing problem or both,
340 persons had symptom of fever or breathing problem or both,

30 persons had all three symptoms (fever, cough and breathing problem).
If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is ________.

Solution

Physics

1.

(c) X experiences extension and Y compression

Explanation:

Let X rod make an angle θ with Y rod in horizontal. Let tension in X rod be T_1 and that in Y rod be T_2, Since the system is in equilibrium. Resolving the forces in x (horizontal) and y (vertical) directions. Assuming tensile forces to be positive and compressive forces negative.

$T_1 \cos\theta + T_2 = 0$

$T_1 \sin\theta = Mg$

$T_1 = \frac{Mg}{\sin @}$

$T_2 = -Mg\cot\theta$

X rod experiences extension and Y rod experiences compression as negative.

2.

(d) $M = \sqrt{L_1 L_2}$

Explanation:

For the first solenoid:

$L_1 = \frac{\mu_0 N_1^2 A}{l}$ ...(i)

For the second solenoid:

$L_2 = \frac{\mu_0 N_2^2 A}{l}$ ...(ii)

However, mutual inductance M is given by:

$M = \frac{\mu_0 N_1 N_2 A}{l}$ ...(iii)

Multiplying equation (i) with equation (ii), we get;

$L_1 L_2 = \frac{\mu_0^2 N_1^2 N_2^2 A^2}{l^2}$ ...(iv)

Squaring equation (iii), we get;

$M^2 = \frac{\mu_0^2 N_1^2 N_2^2 A^2}{l^2}$ ...(v)

Combining equation (iv) and (v), we get;

$M^2 = L_1 L_2$ or $M = \sqrt{L_1 L_2}$

3.

(d) decreases

Explanation:

Entropy is a measure of disorder. When water is converted to ice, disorder decreases, so entropy decreases.

4.

(b) $v \propto x^{-1/2}$

Explanation:

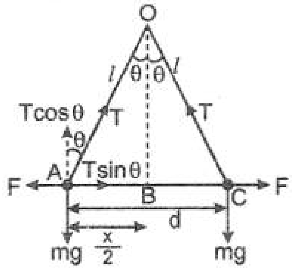


Figure shows equilibrium positions of the two sphere.

$\therefore$ T cosθ = mg and T sinθ = F = $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$

$\therefore \tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2 mg}$

When charge begins to leak from both the spheres at a constant rate, then

$\tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2 mg}$

$\frac{x}{2l} = \frac{1q^2}{4\pi\varepsilon_0 x^2 mg} \quad \left(\because \tan\theta = \frac{x}{2l}\right)$

or $\frac{x}{2l} \propto \frac{q^2}{x^2}$

or $q^2 \propto x^3 \Rightarrow q \propto x^{3/2}$

$\frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt}$

or $v \propto x^{-1/2}$ ($\because \frac{dq}{dt}$ = constant)

5. **(a)** p-type semiconductors

(d) intrinsic semiconductors

Explanation: Holes are electron vacancies which participate in electrical conductivity. These are produced in semiconductors. In n-type semiconductor electrons are the charge carriers.

6. **(a)** The filament consumes less electrical power towards the end of the life of the bulb

(c) The filament emits more light at higher band of frequencies before it breaks up

Explanation: With the use of filament and the evaporation involved, the filament will become thinner thereby decreasing the area of cross-section and increasing the resistance. Therefore the filament will consume less power towards the end of life.

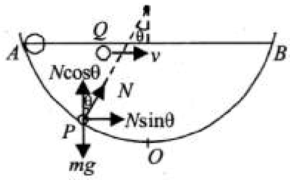
As power, $P = \frac{V^2}{R}$ or, $P \propto \frac{1}{R}$ ($\because$ V = constant)

As the evaporation is non-uniform, the area of cross-section will be different at different cross-section. Therefore temperature distribution will be non-uniform. The filament will break at the point where the temperature is maximum.

When the filament temperature is higher $\left(\lambda_n \propto \frac{1}{T}\right)$, it emits light of lower wavelength or higher band of frequencies.

7. **(b)** $t_P < t_Q$

Explanation: According to question, at A the horizontal speeds of both the masses is the same.



As no force is acting in the horizontal direction the velocity of Q remains the same in horizontal.

In case of P as shown in figure at any intermediate position, the horizontal velocity first increases due to N sinθ, reaches a max value at O and then decreases. But, it always remains greater than v. So, $t_P < t_Q$.

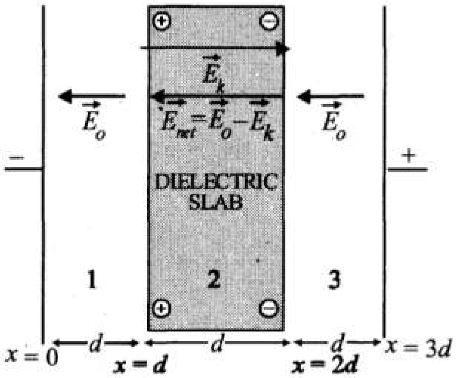
8. **(b)** the direction of the electric field remains the same.

(d) the electric potential increases continuously

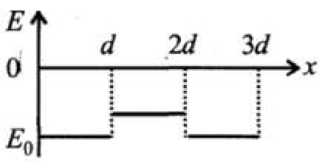
Explanation:

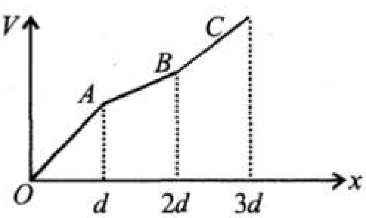
a. In region 1 and 3 , there will be electric field $\vec{E}_0$ directed from positive to negative. In region 2, due to orientation of dipoles, there is an electric field $\vec{E}_k$ present in opposite direction of $\vec{E}_0$. But since $\vec{E}_0$ is also present, the net electric field is $\vec{E}_0$ - $\vec{E}_k$ in the direction of $\vec{E}_0$ as shown in the diagram.

$(\because E_0 > E_k)$



b. The direction of electric field remains the same. In region-2 electric field is less an compared to region 1 and 3.





c. When one moves opposite to the direction of electric field, the potential always increases. The stronger the electric field, the more is the potential increase. Since in region 2, the electric field is less as compared to 1 and 3 therefore the increase in potential will be less but there has to be increase in potential in all the regions from x = 0 to x = 3d.

Hence option (the electric potential increases continuously.) is correct and (the electric potential increases at first, then decreases and again increases) is incorrect.

9. 3

Explanation:

Let $d \propto \rho^x S^y f^z$ $K\rho^x S^y f^z$ where K is dimensionless constant.

$M^0L^1T^0 = M^xL^{-3x}M^yT^{-3y}T^{-z}$

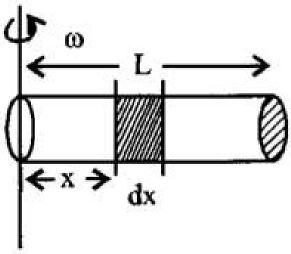
Equating dimensions both sides

$M^0L^1T^0 = M^{x+y}L^{-3x}T^{-3y-z}$

$\therefore$ x + y = 0, -3x = 1 $\therefore x = -\frac{1}{3}$ and y = $\frac{1}{3}$ $\therefore$ n = 3

10. 4

Explanation:



The force exerted by the liuqid at the other end,

$F = \int(dm)\omega^2 x = \int_0^L \left(\frac{m}{L}dx\right)\omega^2 x$ (for small element dm = $\frac{mdx}{L}$)

$= \frac{m}{l}\omega^2\frac{L^2}{2} = \frac{m\omega^2 L}{2}$

The angular velocity will be

$\omega = \sqrt{\frac{2}{mL}}\sqrt{F} = \sqrt{\frac{2F}{0.25\times0.5}} = \sqrt{16}\sqrt{F}$

$\omega = 4\sqrt{F}$ rad s^{-1}

Compair with $\omega = x\sqrt{F}$ so, x = 4

11. 2

Explanation:

Average speed along x-axis, $v_x = \frac{d_1+d_2}{t_1+t_2}$

Here, $r_1 = \frac{mv_0}{qB_1}$ and $r_2 = \frac{mv_0}{qB_2}$

$\therefore B_1 = \frac{B_2}{4}$

$\therefore r_1 = 4r_2$

Time spent by charged particle in B_1, $t_1 = \frac{\pi m}{qB_1}$

Time spent by charged particle in B_2, $t_2 = \frac{\pi m}{qB_2}$

Total distance along x-axis $d_1 + d_2 = 2r_1 + 2r_2 = 2(r_1 + r_2) = 2(5r_2) = 10r_2$

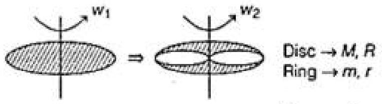
Average speed $= \frac{10r_2}{5t_2} = 2\frac{mv_0}{qB_2} \times \frac{qB_2}{\pi m}$

$= 2ms^{-1}$ ($\because v_0 = \pi ms^{-1}$)

12. 8

Explanation:

$I_1\omega_1 = I_2\omega_2$



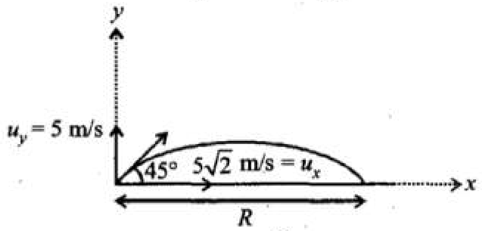
$\therefore \omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left[\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2+2(mr^2)}\right]\omega_1$

$= \left[\frac{50(0.4)^2}{50(0.4)^2+8\times(6.25)\times(0.2)^2}\right](10) = 8$ rad/s

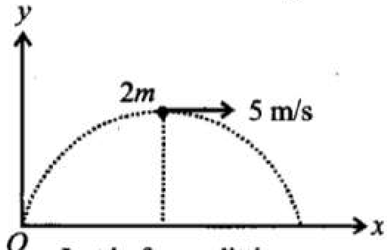
13. 7.50

Explanation:

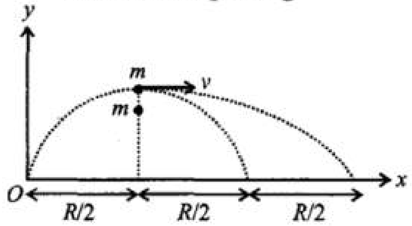
Range $R = \frac{2u_xu_y}{g} = \frac{2\times5\times5}{10} = 5$ m



Time of flight $T = \frac{2u_y}{g} = \frac{2\times5}{10} = 1$ second



Just before splitting



Just after splitting

$\because$ Time of motion of one part falling vertically down-words = 0.5sec $= \frac{T}{2}$

From low coservation of momentum, $P_i = P_f$

$2m \times 5 = m \times v \Rightarrow v = 10$m/s

As there is no initial velocity in vertical direction for second particle, so it will also take same time as first one.

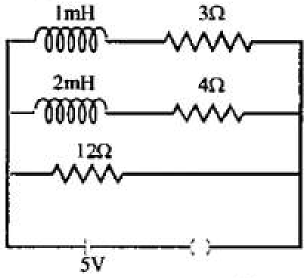
$\therefore$ Time of motion of another part, $t = \frac{T}{2} = 0.5$ sec

Displacement of other part in 0.5 sec in horizontal direction = $v\frac{T}{2}$ = 10 × 0.5 = 5m = R

∴ Total distance of second part from point 'O' is x = $\frac{3R}{2}$ = 3 × $\frac{5}{2}$ ⇒ x = 7.5 m

14. 8

Explanation:



At t = 0 $I_{\min} = \frac{5}{12}$

At t = ∞

$I_{\max} = \frac{5}{R_{eq}} = \frac{5}{\frac{3}{2}} = \frac{10}{3}$

$\left[\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12}\right]$

$\therefore \frac{I_{\max}}{I_{\min}} = \frac{10}{3} \times \frac{12}{5} = 8$

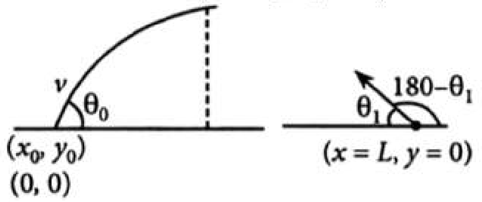
15. 2

Explanation:

Relative acceleration $= 0$

For collision, $v_0 \sin\theta_0 = v\sin\theta_1$ ...(i)

Time, $\Delta t = \frac{L}{v_0\cos\theta_1 + v_0\cos\theta_0}$ ...(ii)



Case I : $(\theta_0, \theta_1) = (45^\circ, 45^\circ), v = v_0$

$\Delta t_1 = T_1 = \frac{L}{v_0(\cos 45^\circ + \cos 45^\circ)} = \frac{L}{\sqrt{2}v_0}$ (Using (ii))

Case II : $(\theta_0, \theta_1) = (60^\circ, 30^\circ), v = \sqrt{3}v_0$ $\left(\text{Using } (i), v_0 \sin 60^\circ = v \sin 30^\circ\right)$

$\Delta t_2 = T_2 = \frac{L}{v_0\cos 60^\circ + v\cos 30^\circ} = \frac{L}{\frac{v_0}{2} + \frac{3v_0}{2}} \therefore T_2 = \frac{L}{2v_0}$

$\Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{L \times 2v_0}{\sqrt{2}v_0.L}\right)^2 = 2$

16. 6

Explanation:

By mass conservation

$\rho\frac{4}{3}\pi R^3 = \rho K\frac{4}{3}\pi r^3$

$\therefore R = K^{1/3}r$ or $r = K^{-1/3}R$

$\therefore \Delta U = T\Delta A = T\left[K4\pi r^2 - 4\pi R^2\right]$

$= T\left[K \cdot 4\pi R^2 K^{-2/3} - 4\pi R^2\right]$

$= 4\pi R^2 T\left[K^{+1/3} - 1\right]$

Putting the values :

$10^{-3} = \frac{10^{-1}}{4\pi} \times 4\pi \times 10^{-4}\left[K^{+1/3} - 1\right]$

$\therefore \quad K^{+1/3} - 1 = 100$

or $K^{+1/3} \cong 100$

Given that, $K = 10^{\alpha}$

$\therefore \quad 10^{+\alpha/3} = 10^2$

$\therefore \quad \frac{\alpha}{2} = 2$

or $\alpha = 6$

Chemistry

17.

(b) Y will oxidise X and not Z

Explanation:

Higher the value of reduction potential, stronger the oxidising agent.

$\because E^0 : Z > Y > X$

$\Rightarrow$ Y will oxidise X but not Z

18. **(a)** 1.5

Explanation:

The van der Waals' equation of state is $\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$

For one mole and when b = 0, the above equation condenses to $\left(p + \frac{a}{V^2}\right)V = RT$

$\Rightarrow \quad pV = RT - \frac{a}{V}$ ...(i)

Eq. (i) is a straight equation between pV and $\frac{1}{v}$ whose slope is ' - a' Equating with slope of the straight line given in the graph.

$-a = \frac{20.1 - 21.6}{3 - 2} = -1.5$

$\Rightarrow$ a = 1.5

19.

(b) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

Explanation:

$\Delta H = \Delta E + \Delta nRT$ For $\Delta H \neq \Delta E$, $\Delta n \neq 0$

Where Δn = no. of moles of gaseous products - no. of moles of gaseous reactants

i. Δn = 2 - 2 = 0

ii. Δn = 0 ($\because$ they are either in solid or liquid state)

iii. Δn = 1 - 1 = 0 ($\because$ C is in solid state)

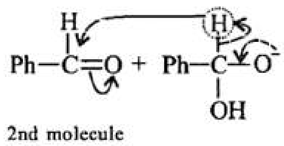
iv. Δn = 2 - 4 = -2

20. **(a)** the transfer of hydride to the carbonyl group

Explanation:

The possible mechanism is

$$\text{i. } Ph - \overset{H}{\overset{|}{C}} = O + OH^{-} \rightleftharpoons ph - \overset{H}{\overset{|}{\underset{OH}{\underset{|}{C}}}} - O^{-}$$

ii. 

$$\xrightarrow{slow} ph - \overset{H}{\overset{|}{\underset{OH}{\underset{|}{C}}}} - O^{-} + Ph - \underset{OH}{\underset{|}{C}} = O$$

$$\rightarrow \text{Ph-CH}_2\text{-OH} + Ph - \underset{O_{-}}{\underset{|}{C}} = O$$

21. **(a)** $[Co(NH_3)_4Cl_2]^+$ and $[Pt(NH_3)_2(H_2O)Cl]^+$

(c) $[Pt(NH_3)_3(NO_3)]Cl$ and $[Pt(NH_3)_3Cl]Br$

Explanation: The pair of complex ions $[Co(NH_3)_4Cl_2]^+$ and complex ions $[Co(NH_3)_4Cl_2]^+$ and $[Pt(NH_3)_2(H_2O)Cl]^+$ show geometrical isomerism.

The pair of complexes $[Pt(NH_3)_3(NO_3)Cl$ and $[Pt(NH_3)_3Cl]Br$ show ionisation isomerism. The other pairs given do not have same type of isomerism.

The complex $[CoBr_2Cl_2]^{2-}$ will show sp^3 hybridisation. The tetrahedral shape will not show any kind of isomerism.

(c) The complex $[ptBr_2Cl_2]^{2-}$ will be show dsp^2 hybridisation. The square planar shape will show geometrical isomerism.

Hence, (c) is not an answer.

Note: Br and Cl are weak filed ligand but in this case they will cause pairing of electrons. Most of the platinum complexes with four ligand are square planar in shape.

22. **(a)** $\Delta H = 0$

(c) ΔS_{system} is positive

Explanation: For ideal solution, $\Delta S_{\text{system}} > 0; \Delta H_{\text{mixin}} = 0$ Since, there is no heat exchange between system and surrounding;

$\Delta S_{\text{surroundings}} = 0 \times \Delta G = \Delta H - T\Delta S_{\text{system}} < 0.$

23. **(a)** I is physisorption and II is chemisorption

(c) IV is chemisorption and II is chemisorption

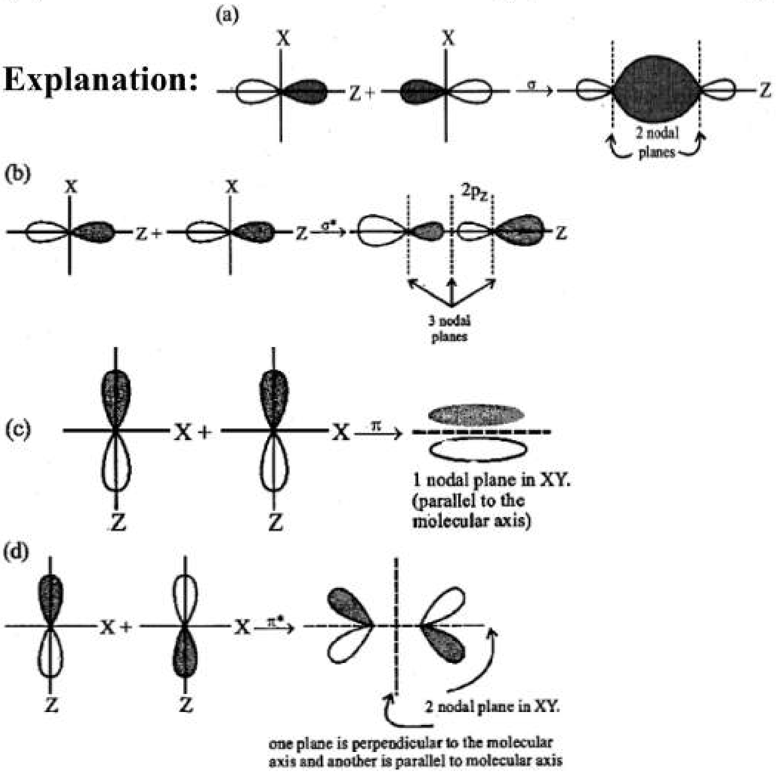
Explanation: Graph (I) and (HI) represent physisorption because, in physisorption, the amount of adsorption decreases with the increase of temperature and increases with the increase of pressure.

Graph (II) represent chem isorption because in chemisorption amount of adsorption increases with the increase of temperature. Graph (IV) is showing the formation of a chemical bond, hence chemisorption.

24. **(b)** σ orbital has a total of two nodal planes.

(d) π^* orbital has one node in the xy-plane containing the molecular axis.

Explanation:



25. 15.05

Explanation:

In pure iron oxide (FeO), iron and oxygen are present in the ratio 1 : 1

However, here number of Fe^{2+} present = 0.93 or No. of Fe^{2+} ions missing = 0.07

Since each Fe^{2+} ion has 2 positive charge, the total number of charge due to missing (0.07) Fe^{2+} ions $= 0.07 \times 2 = 0.14$. To maintain electrical neutrality, 0.14 positive charge is compensated by the presence of Fe^{3+} ions. Now since the replacement of one Fe^{2+} ion by one Fe^{3+} ion increases one positive charge, 0.14 positive charge must be compensated by the presence of 0.14 Fe^{3+} ions.

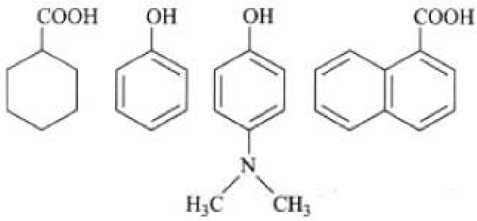
In short, 0.93 Fe^{2+} ions have 0.14 Fe^{3+} ions.

100 Fe^{2+} ions have $= \frac{0.14}{0.93} \times 100 = 15.05\%$

26. 4

Explanation:

Aromatic alcohols and carboxylic acids forms salt with NaOH, will dissolve in aqueous NaOH are as follows:



27. 6

Explanation:

$H_2SO_4(aq) \longrightarrow \underset{1M}{H^+}(aq) + \underset{1M}{HSO_4^-}(aq)$

(as is very large)

$Na_2SO_4(aq) \longrightarrow 2Na^+(aq) + \underset{0.018M}{SO_4^{2-}}(aq)$

(as Na_2SO_4 is a salt)

Now, due to common ion effect, dissociation of HSO_4^- will be suppressed.

$\underset{1M}{HSO_4^-}(aq) \rightleftharpoons \underset{1M}{H^+}(aq) + \underset{0.018M}{SO_4^{2-}}(aq) \quad (at\ start)$

$Q_C = \frac{[H^+][SO_4^{2-}]}{[HSO_4^-]} \Rightarrow Q_C = \frac{1\times0.018}{1} = 0.018$

$K_C = K_{a_2} = 1.2 \times 10^{-2} = 0.012$

Thus, $Q_C > K_C$ and the reaction will move in backward direction.

$\underset{(1+x)}{HSO_4^-}(aq)\underset{(1-x)}{H^+}(aq) + \underset{(0.018-x)}{SO_4^{2-}}(aq)$

$K_{a_2} = \frac{(1-x)(0.018-x)}{(1+x)} = 0.012$

Assuming x to be very small, 1 - x ≈ 1 and 1 + x ≈ 1.

$\Rightarrow 0.018 - x = 0.012 \Rightarrow x = 0.006$

$[SO_4^{2-}] = 0.018 - x = 0.012$ mol/L

$\underset{1M}{HSO_4^-}(aq) \rightleftharpoons \underset{1M}{H^+}(aq) + \underset{0.018M}{SO_4^{2-}}(aq) \quad (at\ start)$

$K_{sp} = S(S + 0.012)$

Since, S is very small, $S + 0.012 \approx 0.012$

$\Rightarrow 1.6 \times 10^{-8} = S \times 0.012$

$\Rightarrow S = 1.33 \times 10^{-6} = X \times 10^{-Y} \Rightarrow Y = 6$

28. 10.0

Explanation:

This question is incomplete, order is not given. If we consider, the given reaction to be first order, then

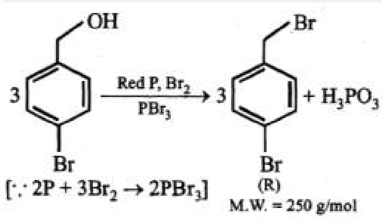
$t_{1/2} = \frac{\ln 2}{k} \Rightarrow k = \ln 2$

$\therefore t = \frac{1}{k} \ln \frac{100}{100-99.9} \Rightarrow t = \frac{1}{\ln 2} \ln 10^3$

$\Rightarrow t = \frac{1\times3\times\ln 10}{\ln 2} = \frac{3\times2.3}{0.69} = 10$ min.

29. 1.5

Explanation:



In Carius method, the halogen present in the organic compound completely forms the silver halide.

No. of moles of (R) = $\frac{1g}{250g/mol} = \frac{1}{250}$ mol of AgBr

1 mol of (R) ≡ 2 mol of ≡ 2 mol of AgBr

$\therefore \frac{1}{250}$ mol of (R) ≡ $\frac{2}{250}$ mol of Br ≡ $\frac{2}{250}$ mol of AgBr

M.W. of AgBr = 108 + 80 = 188 g/mol

$\therefore$ The amount of AgBr formed = $\frac{2\times188}{250}$ = 1.50 g

30. 30

Explanation:

For single electron system, $r_n = 52.9 \times \frac{n^2}{Z}$pm

$105.8 = \frac{52.9\times n_1^2}{2}$ $\therefore n_2 = 4 \Rightarrow n_1 = 2$

$26.45 = \frac{52.9\times n_2^2}{2}$ $\therefore n_2 = 1$

So, transition is from 2 to 1.

Now $\frac{hc}{\lambda} = R_H Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\lambda = 30 \times 10^{-9}$m = 30nm

31. 10.43

Explanation:

Molality = $\frac{\text{Mass of solute / M. wt. of solute}}{\text{Mass of solvent in kg}}$

Mass of H_2SO_4 in 100 ml of 93% H_2SO_4 solution = 93 g

$\therefore$ Mass of H_2SO_4 in 1000 ml of the H_2SO_4 solution = 930 g

Mass of 1000 ml H_2SO_4 solution = 1000 × 1.84 = 1840 g

Mass of water in 1000 ml of solution = 1840 - 930 g

= 910 g = 0.910 kg

Moles of H_2SO_4 = $\frac{\text{Wt. of } H_2SO_4}{\text{Mol. wt. of } H_2SO_4} = \frac{930}{98}$

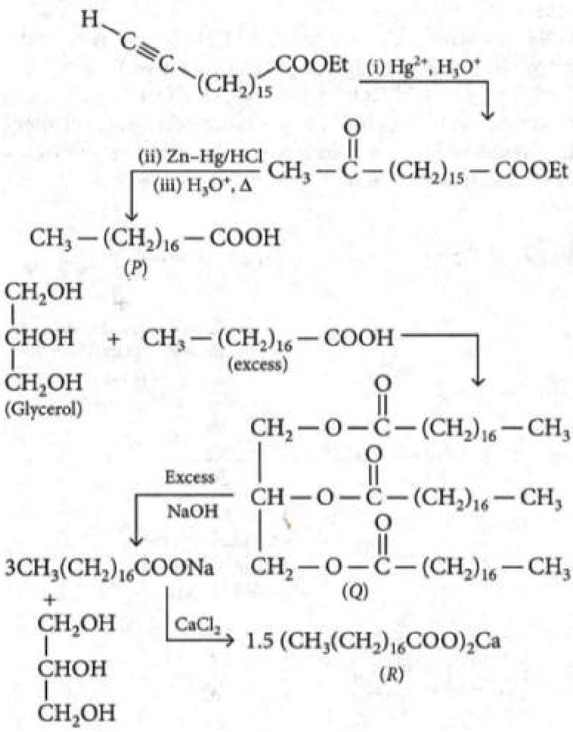
$\therefore$ Moles of H_2SO_4 in 1 kg of water

$= \frac{930}{98} \times \frac{1}{0.910}$ = 10.43 mol

$\therefore$ Molality of solution = 10.43 m

32. 909

Explanation:



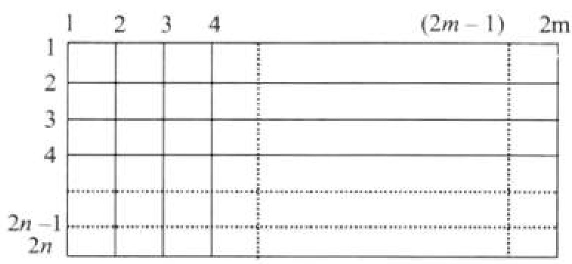
The amount of R produced in gram is 1.5 x M.W. of R = 1.5[{(18 x 12) + (16 x 2) + 3 + 32} x 2 + 40] g = (1.5 x 606) g = 909 g

Mathematics

33.

(d) m^2n^2

Explanation:



If we see the blocks in terms of lines then there are 2m vertical lines and 2n horizontal lines. To form the required rectangle, we must select two horizontal lines, one even-numbered (out of 2, 4, 2n) and one odd-numbered (out of 1, 3, ..., 2n - 1) and similarly two vertical lines. The number of rectangles

$= {}^mC_1 \times {}^mC_1 \times {}^nC_1 \times {}^nC_1 = m^2n^2$

34.

(d) $t_4 > t_3 > t_1 > t_2$

Explanation:

As when $\theta \in \left(0, \frac{\pi}{4}\right)$, $\tan\theta < \cot\theta$

Since, $\tan\theta < 1$ and $\cot\theta > 1$

$\therefore (\tan\theta)^{\cot\theta} < 1$ and $(\cot\theta)^{\tan\theta} > 1$

$\therefore t_4 > t_1$ which only holds.

35.

(d) A.P

Explanation:

According to the given condition, $b^2 = ac$

$\Rightarrow$ Equation $ax^2 + 2bx + c = 0$ can be written as

$\Rightarrow ax^2 + 2\sqrt{ac}\ x + c = 0$

$\Rightarrow (\sqrt{a}x + \sqrt{c})^2 = 0 \Rightarrow x = -\sqrt{\frac{c}{a}}$ (repeated root)

This must be the common root by hypothesis. So it must satisfy the equation $dx^2 + 2ex + f = 0$

$\Rightarrow d\left(\frac{c}{a}\right) - 2e\sqrt{\frac{c}{a}} + f = 0$

$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{c}\sqrt{\frac{c}{a}} = \frac{2e}{b}$

$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

36.

(c) a hyperbola

Explanation:

Given equation is $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1$, where $|r| < 1$

$\Rightarrow$ 1 - r is (+ve) and 1 + r is (+ve)

$\therefore$ Given equation is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Hence, it represents a hyperbola when $|r| < 1$.

37. **(a)** $\lim\limits_{x\to1^+} f(x)$ does not exist

(d) $\lim\limits_{x\to1^-} f(x) = 0$

Explanation: Given: $f(x) = \frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$

$\lim\limits_{x\to1^-} f(x) = \lim\limits_{h\to0} \frac{1-(1-h)(1+h)}{h}\cos\left(\frac{1}{h}\right)$

$= \lim\limits_{h\to0} \frac{1-1+h^2}{h}\cos\left(\frac{1}{h}\right) = \lim\limits_{h\to0} h\cos\left(\frac{1}{h}\right) = 0$

$\lim\limits_{x\to1^+} f(x) = \lim\limits_{h\to0} \frac{1-(1+h)(1+h)}{h}\cos\left(\frac{1}{h}\right)$

$= \lim\limits_{h\to0} \frac{-2h-h^2}{h}\cos\left(\frac{1}{h}\right) = \lim\limits_{h\to0}(-2-h)\cos\left(\frac{1}{h}\right)$

$= -2 \times$ (Some value oscillating between -1 and 1)

$\therefore \lim\limits_{x\to1^+} f(x)$ does not exist.

38. **(a)** there is 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(b) determinant of $(M^2 + MN^2)$ is 0

Explanation: Given: $MN = NM$, $M \neq N^2$ and $M^2 = N^4$.

Then $M^2 = N^4 \Rightarrow (M + N^2)(M - N^2) = 0$

$\Rightarrow$ (i) $M + N^2 = 0$ and $M - N^2 \neq 0$

(ii) $|M + N^2| = 0$ and $|M - N^2| = 0$

In each case $|M + N^2| = 0$

$\therefore |M^2 + MN^2| = |M|\,|M + N^2| = 0$

$\therefore$ determinant of $(M^2 + MN^2)$ is 0 is correct and determinant of $(M^2 + MN^2) \geq 1$ is not correct.

Also we know if $|A| = 0$, then there can be many matrices U, such that $AU = 0$

$\therefore (M^2 + MN^2)U = 0$ will be true for many values of U.

$\therefore$ here is 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix is correct.

Again if $AX = 0$ and $|A| = 0$, then X can be non-zero.

$\therefore$ for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix is not correct.

39. **(a)** 58

(b) 55

(c) 56

Explanation: For n = 3, $P = \begin{bmatrix} w^2 & w^3 & w^4 \\ w^3 & w^4 & w^5 \\ w^4 & w^5 & w^6 \end{bmatrix}$

and $P^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

It shows $P^2 = 0$ if n is a multiple of 3

So for $P^2 \neq 0$, n should not be a multiple of 3, hence n can take values 55, 58 and 56.

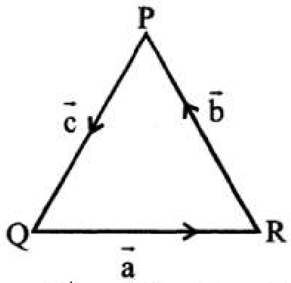
40. **(a)** $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$

(b) $\vec{a}.\vec{b} = -72$

(c) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

Explanation: From given information

$\vec{a} + \vec{b} + \vec{c} = \vec{o}$



$\Rightarrow |\vec{b} + \vec{c}|^2 = |-\vec{a}|^2 \Rightarrow |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c} = |\vec{a}|^2$

$\Rightarrow 48 + |\vec{c}|^2 + 48 = 144 \Rightarrow |\vec{c}|^2 = 48 \Rightarrow |\vec{c}| = 4\sqrt{3}$

$\therefore \frac{|\vec{c}|^2}{2} - |\vec{a}| = \frac{48}{2} - 12 = 12$

$\therefore \frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$ is correct

$\frac{|\vec{c}|^2}{2} + |\vec{a}| = 24 \neq 30$

$\therefore \frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$ is not correct

Also $|\vec{b}| = |\vec{c}| \Rightarrow \angle Q = \angle R$

and $\cos(180 - P) = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}||\vec{c}|} = \frac{1}{2}$

$\Rightarrow \angle P = 120^\circ \therefore \angle Q = \angle R = 30^\circ$

And $\vec{a}\vec{b} = 12 \times 4\sqrt{3} \times \cos 150 = -72$

$\therefore \vec{a}.\vec{b} = -72$ is correct

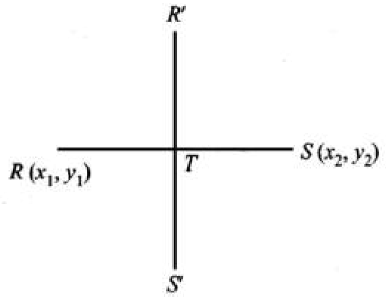
Now, $\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{0} \Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$

$\therefore |\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 2|\vec{a} \times \vec{b}| = 2 \times 12 \times 4\sqrt{3} \times \sin 150 = 48\sqrt{3}$

$\therefore |\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$ is correct.

41. 77.14

Explanation:



Perpendicular bisector of RS

$T \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Since, $x_1 + x_2 = 0 \Rightarrow y_1 + y_2 = 2$

So, T = (0, 1) [from (i)]

Equation of R'S' :

$(y - 1) = -\frac{1}{2}(x - 0) \Rightarrow x + 2y = 2$

Let $R'(a_1, b_1)$ and $S'(a_2, b_2)$

$\therefore D = (a_1 - a_2)^2 + (b_1 - b_2)^2 = 5(b_1 - b_2)^2$

On solving $x + 2y = 2$ and $|2x^2 - (y - 1)^2 = 3\lambda^2$, we get

$\Rightarrow |8(y - 1)^2 - (y - 1)^2| = 3\lambda^2$

$\Rightarrow (y-1)^2 = \left(\frac{\sqrt{3}\lambda}{\sqrt{7}}\right)^2$

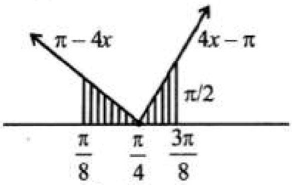
$y - 1 = \pm\frac{\sqrt{3}\lambda}{\sqrt{7}}$

$\Rightarrow b_1 = 1 + \frac{\sqrt{3}\lambda}{\sqrt{7}}, b_2 = 1 - \frac{\sqrt{3}\lambda}{\sqrt{7}}$

$D = 5\left(\frac{2\sqrt{3}\lambda}{\sqrt{7}}\right)^2 = \frac{5\times4\times3\lambda^2}{7} = \frac{5\times4\times27}{7} = 77.14$

42. 1.5

Explanation:



$$S_2 = \int_{\pi/8}^{3\pi/8} f(x) g_2(x) dx$$

$$= \int_{\pi/8}^{3\pi/8} \sin^2 x|4x - \pi| dx \text{ ...(i)}$$

$$= \int_{\pi/8}^{3\pi/8} \sin^2\left(\tfrac{\pi}{8} + \tfrac{3\pi}{8} - x\right)\left|4\left(\tfrac{\pi}{8} + \tfrac{3\pi}{8} - x\right) - \pi\right| dx$$

$$= \int_{\pi/8}^{3\pi/8} \left(\cos^2 x\right)|\pi - 4x| dx$$

$$= \int_{\pi/8}^{3\pi/8} \cos^2 x|4x - \pi| dx \text{ ...(ii)}$$

Adding equations (i) and (ii) we get

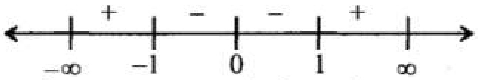
$$2S_2 = \int_{\pi/8}^{3\pi/8} |4x - \pi| \left(\sin^2 x + \cos^2 x\right) dx = \int_{\pi/8}^{3\pi/8} |4x - \pi| dx$$

$$= 2 \times \tfrac{1}{2} \times \tfrac{\pi}{8}$$

$$\Rightarrow \tfrac{48S_2}{\pi^2} = \tfrac{3}{2} = 1.5$$

43. 4.0

Explanation:



$3x^2 + x - 1 = 4\left|x^2 - 1\right|$

Case 1: If $x \in [-1, 1]$

$3x^2 + x - 1 = -4x^2 + 4$

$\Rightarrow 7x^2 + x - 5 = 0 \because D = 141 > 0$

$\therefore$ Equation has two roots

Case 2: If $x \in (-\infty, -1] \cup [1, \infty)$

$3x^2 + x - 1 = 4x^2 - 4$

$\Rightarrow x^2 - x - 3 = 0 \because D = 13 > 0$

$\therefore$ Equation has two roots So, total 4 roots

44. 4

Explanation:

We have, $S_k = \frac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$

Now, $(k^2 - 3k + 1)S_k = \{(k - 2)(k - 1) - 1\} \times \frac{1}{(k-1)!}$

$= \frac{1}{(k-3)!} - \frac{1}{(k-1)!}$

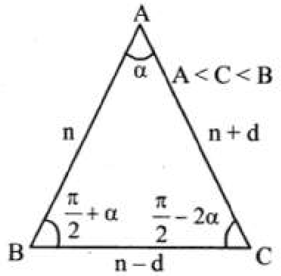
$\Rightarrow \sum_{k=1}^{100} \left|\left(k^2 - 3k + 1\right) S_k\right| = 1 + 1 + 2 - \left(\frac{1}{99!} + \frac{1}{98!}\right) = 4 - \frac{100^2}{100!}$

$\Rightarrow \frac{100^2}{100!} + \sum_{k=1}^{100} \left|\left(k^2 - 3k + 1\right) S_k\right| = 4$

45. 1008.0

Explanation:

Let sides of triangle are n - d, n, n + d.



$\mathrm{n} - \mathrm{d} = 2\sin\alpha$ ...(i)

$\mathrm{n} + \mathrm{d} = 2\sin\left(\frac{\pi}{2} + \alpha\right)$

$\Rightarrow \mathrm{n} + \mathrm{d} = 2\cos\alpha$ ...(ii)

$\mathrm{n} = 2\sin\left(\frac{\pi}{2} - 2\alpha\right)$

$\Rightarrow \mathrm{n} = 2\cos 2\alpha$ ...(iii)

Adding (i) and (ii), we get $\mathrm{n} = \sin\alpha + \cos\alpha$

$\Rightarrow 2\cos 2\alpha = \sin\alpha + \cos\alpha$

$2\left(\cos^2\alpha - \sin^2\alpha\right) - (\cos\alpha + \sin\alpha) = 0$

$(\cos\alpha + \sin\alpha)[2(\cos\alpha - \sin\alpha) - 1] = 0$

$\Rightarrow 2(\cos\alpha - \sin\alpha) = 1 \Rightarrow \sin 2\alpha = \frac{3}{4}$

Then, $a = \frac{1}{2}.\mathrm{n}.(\mathrm{n} + \mathrm{d})\cdot\sin\alpha = \frac{1}{2}\cdot 2\cos 2\alpha\ 2\cos\alpha\cdot\sin\alpha$

$=\sin 2\alpha\cdot\cos 2a$

$= \frac{3}{4} \times \frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{16}$

$(64a)^2 = \left(64 \times \frac{3\sqrt{7}}{16}\right)^2 = 16 \times 9 \times 7 = 1008$

46. 1

Explanation:

$f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$

$= \sin\left[\sin^{-1}\left(\frac{\sin\theta}{\sqrt{\sin^2\theta + \cos 2\theta}}\right)\right] \left[\because \tan^{-1}\frac{x}{y} = \sin^{-1}\frac{x}{\sqrt{x^2+y^2}}\right]$

$= \sin\left[\sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)\right] = \tan\theta$

$\Rightarrow \frac{df(\theta)}{d\tan\theta} = 1$

47. 0.75

Explanation:

Given that lines are $\vec{r} = \lambda\hat{i}$ ...(i)

$\vec{r} = \mu(\hat{i} + \hat{j})$ ...(ii)

$\vec{r} = v(\hat{i} + \hat{j} + \hat{k})$ ...(iii)

These lines cut the plane x + y + z = 1 at points A (λ, 0, 0), B (μ, μ, 0) and C (v, v, v) respectively

Since, A lies on plane $\Rightarrow \lambda = 1 \Rightarrow$ A (1, 0, 0)

Since, B lies on plane $\Rightarrow \mu + \mu = 1 \Rightarrow \mu = \frac{1}{2}$

$\Rightarrow B\left(\frac{1}{2}, \frac{1}{2}, 0\right)$

Since, C lies on plane $\Rightarrow$ v + v + v = 1 $\Rightarrow v = \frac{1}{3}$

$\Rightarrow C\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

Area (Δ ABC) = $\frac{1}{2}|\overline{AB} \times \overline{AC}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix}$

$= \frac{1}{2}\left|\frac{1}{6}\hat{i} + \frac{1}{6}\hat{j} + \frac{1}{6}\hat{k}\right| = \frac{1}{2} \times \frac{1}{6}\sqrt{3} = \frac{\sqrt{3}}{12}$

$\therefore (6\Delta)^2 = 36 \times \frac{3}{144} = \frac{3}{4} = 0.75$

48. 0.8

Explanation:

Let F, C and B be the set of people having symptoms of fever, cough and breathing problem respectively.

Here, we have n(F) =190, n(B) = 220 and n(C) = 220

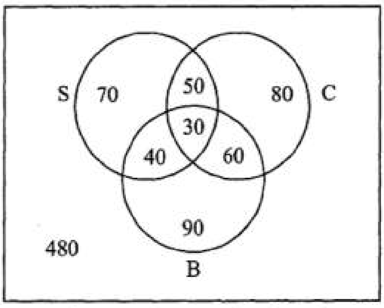
Also, n(F ∪ C) = 330, n(C ∪ B) = 350, n(F ∪ B) = 340 and n(F ∩ C ∩ B) = 30

So n(F ∩ C) = n(F) + n(C) - n(F ∪ C) = 80

Similarly, n(F ∩ B) = 70 and n(C ∩ B) = 90

By Venn diagram, we can better understand.

Here,



Number of people having at most one symptom

$= 70 + 80 + 90 + 480 = 720$

Required probability $= \frac{720}{900} = 0.8$